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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

AN ANALYSIS OF
NAVY INVENTORY MODELS AND
A PROPOSAL FOR NON-AUTOMATED SHIPS

by

Howard Paul Gorman, Jr.

September 1978

Thesis Advisor: F.R. Richards

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An Analysis of
Navy Inventory Models and
A Proposal for Non-Automated Ships

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I. INTRODUCTION

The field of inventory theory is one of the most researched subjects in management science or operations research. Inventory methods have also been considered one of the most successful applications of operations research. The costs and other factors are easy to define in business, and inventory systems are adaptable to automated data processing.

In contrast to the business world, inventory mangement within the Navy is much more difficult and complex. The size of the inventory system creates computational difficulties even with today's high speed, efficient computers. In addition to sheer size, the multi-echelon nature of the system creates further complexities. Inventory is managed at three different levels, each operating a separate inventory system. At the highest level, the Navy's Inventory Control Points (ICP's) procure and position material at approximately seventy world-wide stock points. These stock points operate under the Variable Operating and Safety Level (VOSL) program, providing material to the end users. Finally, at the lowest level is the consumer, principally the shipboard users. Ships with computer capability operate under the Shipboard Uniform Automated Data Processing System (SUADPS), whereas non-automated ships presently use a simple fixed-months-supply system. In addition to the difficulties caused by the size and the multi-echelon nature of Navy inventory, it

is also extremely difficult to define both the costs relevant to the inventory models and the probability distributions of demand. Further, even if the costs and probability distributions were known, the parameters of the distributions are difficult to forecast accurately. A final problem is constraints. In actual practice Navy inventory systems are always constrained by some factor, usually money, and the active constraint dictates a feasible policy that is not necessarily theoretically optimal. These problems inherent in Navy inventory models will be discussed in more detail in Chapter III, after the theoretical background is discussed in Chapter II.

Chapter IV discusses the four current basic Navy inventory models (UICP, VOSL, AUTOMATED AFLOAT, NON-AUTOMATED AFLOAT) and the inherent problems involved with each model. The purpose of Chapter IV is not to criticize current procedures, but rather to point out certain difficulties that exist and to give justification for a new approach to Navy inventory management. Chapter V outlines this new approach, which optimizes inventory policy based only upon predictable information. Chapter V also applies this new approach to the UICP inventory system. Chapter VI describes a recommended change to non-automated afloat inventory management, again relying on this new approach. The recommendations are based on statistics obtained from work done by the author with the computer simulation afloat inventory model at Navy Fleet Material Support Office.

Before proceeding to a more detailed discussion of the inherent difficulties of Navy inventory models, descriptions of the models, and recommended changes, a review of theory is appropriate and is presented in the next chapter.

II. THEORETICAL BACKGROUND

To begin, the following notation will be used throughout this thesis:

K = Total annual variable costs

λ = Average quantity demanded per year

C = Unit cost

I = Inventory holding cost percentage

A = Cost to place an order

π = Time weighted shortage cost

L = Reorder Lead Time

Q = Quantity to order (Q^* = Optimum Q)

r = Reorder point (r^* = Optimum r)

μ = Expected demand during lead time.

The subscript i will denote the i^{th} item in the n item inventory. For example $\lambda_i C_i$ would be the annual dollar value of demands for item i, and $\sum_{i=1}^{n} \lambda_i C_i$ would be the total annual dollar value of demands for the entire n item inventory.

A. CLASSIC MODEL: DETERMINISTIC DEMANDS, NO BACKORDERS

In the classic deterministic model with no stockouts, it is assumed that demands and procurement lead times are constant. Although these assumptions make this model over-simplistic, its ease of computation makes it useful when the number of items being managed in an inventory system

make more complicated models computationally infeasible. In this model the only variable costs are

Annual ordering cost = # of orders per year x cost to order

$$= \frac{\lambda}{Q} \cdot A = \frac{\lambda A}{Q}, \text{ and}$$

Annual holding cost = inventory holding cost percentage x unit cost x average inventory

$$= I \cdot C \cdot \frac{Q}{2} = \frac{ICQ}{2}$$

$$K = \text{annual ordering cost} + \text{annual holding cost} = \frac{\lambda A}{Q} + \frac{ICQ}{2}$$

The optimal (minimal cost) order quantity is found in a straightforward manner using ordinary calculus to be

$$Q^* = \sqrt{\frac{2\lambda A}{IC}}$$

This is the so-called "Wilson Q" or economic order quantity (EOQ). Despite its simplicity a variation of the EOQ equation is still used in many Navy and other military and commercial models. The only other decision variable is r (reorder point). In this simple model r^*= lead time (in years) x λ, since lead time is constant. It will be seen in Chapter IV that automated ships use inventory procedures similar to those described here. This model was discussed because of its usefulness and to familiarize the reader to the basic costs involved in any inventory system. The important stochastic, real-world model will be discussed next.

B. THE STOCHASTIC DEMANDS, TIME-WEIGHTED BACKORDERS MODEL.

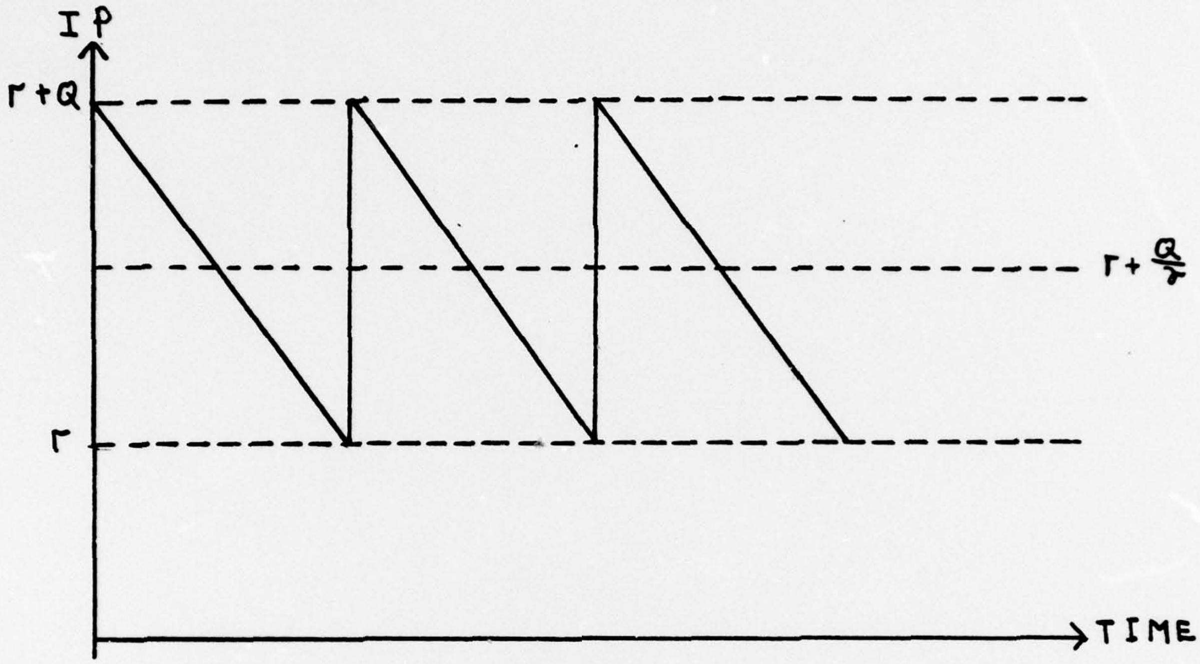
In real-world inventory systems, demands are nearly always random. The classic EOQ model, although sometimes very useful in the real world, cannot be optimal when uncertainty is present. Not only must the random demand distribution be considered in determining the optimum reorder quantity (Q^*), but it is also necessary to include safety stock in the reorder level to provide protection from stockouts during the reorder period.

In this model three costs need to be considered: expected ordering costs, holding costs, and backorder costs. The expected ordering costs determination is straightforward and identical to the corresponding expression in the deterministic model:

$$\text{Expected annual ordering cost} = \frac{\lambda A}{Q} .$$

The holding cost portion of the total expected variable cost equation is slightly more difficult to determine than in the deterministic case. First consider that inventory position (IP) = on-hand quantity (OH) + on-order quantity (OO) - backorders (BO) or OH = IP - OO + BO. Thus expected OH is given by E(OH) = E(IP) - E(OO) + E(BO). Here the assumption is made that the E(BO) term is usually not significant to the total expression and the accuracy lost by dropping the term has little effect on the final decision rules. Dropping the E(BO) term for computational simplicity, the equation becomes

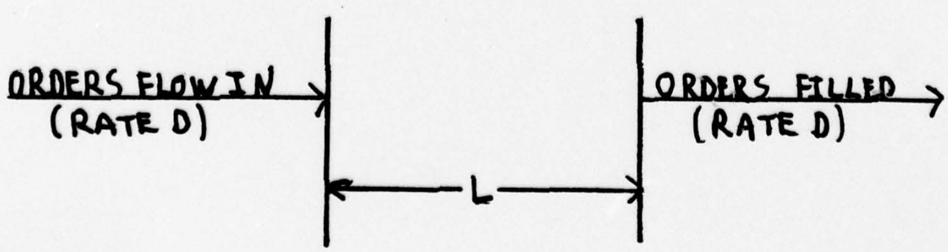
E(OH) = E(IP) - E(OO). The inventory position (OH + OO) of a continuous review (Q,r) inventory system can be described as



Intuitively, the expected inventory position is

$$\frac{1}{2}(r + (r + Q)) = r + \frac{Q}{2} .$$

Finally, E(OO) equals μ, the expected lead time demand. The proof of this equation is difficult to show analytically. Hadley and Whitin [Ref. 7] give the following heuristic treatment:



Orders flow through the system at rate D, and remain in the system for time length L. The expected number in the system is DL which must equal E(OO) in the long run, steady state and by definition E(OO) = μ, and so E(OH) = $r + \frac{Q}{2} - \mu$. Therefore, the expected annual holding cost = $I \cdot C \cdot E(OH)$ $= IC(r + \frac{Q}{2} - \mu)$.

The only remaining cost to evaluate is the expected annual cost of backorders. The method of determining backorder costs depends upon whether these costs are per unit backordered or per unit backordered per unit time (time weighted). DOD Instruction 4140.39 requires military models to consider the latter case and it is assumed in this analysis.

First, assume that demand during lead time L has cumulative distribution function F(x:L) and density f(x:L). As usual, the probability that lead time demand during L is less than or equal to x is F(x:L). If S is the average requisition size and B(Q,r) is the expected number of units backordered at any given time, then $\frac{B(Q,r)}{S}$ is the expected number of requisitions backordered at any given time. It follows that $\frac{\pi}{S} B(Q,r)$ is the expected average annual cost of backordered requisitions. B(Q,r) must now be determined. Consider that inventory position varies between r and r + Q, or that IP is always in a discrete state r + x $(0 \leq x \leq Q)$. The probability of being in any one of these states is $\frac{1}{Q}$ (see Ref. 10). Now consider y to be the potential quantity to be backordered. If y is to be backordered, exactly r + x + y demands must occur during lead time. The probability

of this occuring is $f(r + x + y;L)$. Therefore, the probability $p(y)$ of y items being backordered is

$$p(y) = \frac{1}{Q} \int_0^Q f(y + r + x;L)dx$$

$$= \frac{1}{Q}[F(y + r + Q;L] - F(y + R;L)], \quad y \geq 0$$

To find the total probability of being out of stock, $p(y)$ must be integrated over all values of y.

$$\frac{1}{Q} \int_0^\infty [F(y + r + Q;L) - F(y + r;L)]dy$$

$$= \frac{1}{Q} \int_r^\infty [F(u + Q;L) - F(u;L)]du \; .$$

Thus,

$$B(Q,r) = \frac{1}{Q} \int_r^\infty (x - r)[F(x + Q;L) - F(x;L)]dx.$$

The total expected variable cost equation for the backorders case with time dependent shortage costs is

$$K = \frac{\lambda A}{Q} + IC[r + \frac{Q}{2} - \mu] + \frac{\pi}{S}[B(Q,r)]$$

where F(x;L) is the cumulative distribution of lead time demand. Using ordinary calculus to find the optimum r we have

$$(1) \qquad \frac{\partial K}{\partial r} = IC + \frac{\pi}{S}\frac{\partial B(Q,r)}{\partial r} = 0$$

where

$$\frac{\partial B(Q,r)}{\partial r} = \frac{\partial}{\partial r}\frac{1}{Q}\int_r^\infty (x - r)[F(x + Q;L) - F(x;L)dx$$

$$= \frac{1}{Q}\int_r^\infty (-1)[F(x + Q;L) - F(x;L)dx \,.$$

Substituting this expression into equation (1) gives

$$\frac{\partial K}{\partial r} = IC + \frac{\pi}{SQ}\int_r^\infty (-1)[F(x + Q;L) - F(x;L)]dx = 0$$

or equivalently

$$\int_r^\infty F(x + Q;L) - F(x;L)dx = \frac{ICSQ}{\pi} \,.$$

Defining G(r) to be the integral on the left hand side we arrive at an equation for determining r*

$$(2) \qquad G(r) = \frac{ICSQ}{\pi}.$$

Next, taking the derivative of K with respect to Q and setting it to zero gives

$$\frac{\partial K}{\partial Q} = \frac{-\lambda A}{Q^2} + \frac{IC}{2} + \frac{\pi}{S}\frac{\partial B(Q,r)}{\partial Q} = 0$$

where

$$\frac{\partial B(Q,r)}{\partial Q} = \frac{-1}{Q^2}\int_r^\infty (x - r)[F(x + Q;L) - F(x;L)]dx$$

$$+ \frac{1}{Q}\frac{\partial}{\partial Q}\int_r^\infty (x - r)[F(x + Q;L) - F(x;L)]dx.$$

Due to the complexity of the mathematics, several steps are not shown. The final result is

$$(3) \qquad \frac{\lambda A}{Q} + \frac{\pi}{SQ}\int_r^\infty x[F(x + Q;L) - F(x;L)]dx$$

$$= \frac{ICQ}{2} + ICr + \frac{\pi}{S}\int_r^\infty (x - r)[\frac{\partial F}{\partial Q}(x + Q;L)dx].$$

Equations (2) and (3) are extremely difficult to solve for Q and r, since each equation depends in a complex way on both

unknowns. Even a numerical, iterative procedure would be computationally unwieldy. For example, if one sets Q = Q(Wilson) initially in Eqn. (2), r can be determined if the lead time demand distribution is known. However, using this r to find Q in Eqn. (3) would still be very difficult. In fact, if this time-weighted shortage model is to be used with a large number of inventory items separate techniques must be utilized to determine Q, and equation 2 can then be used to approximate r*.

The theory just presented for both the simple and stochastic models will be used in discussing the current Navy models in Chapter IV. Before addressing those models, the next chapter discusses in more detail some of the inherent problems of Navy inventory models.

III. PROBLEMS INHERENT IN NAVY INVENTORY MODELS

The general procedure outlined above to analytically determine optimum order quantities and reorder points with stochastic demands and time-weighted requisitions backordered can be summarized as follows:

(1) Define the applicable cost equation, including the relevant specific costs.

(2) Use the calculus to determine the equations for the optimum values of the order quantity (how much to order) and the reorder point (when to reorder).

When attempting to apply this procedure to Navy inventory models, numerous problems arise. These problems can be categorized as follows:

(1) Unknown Costs:

A major problem in Navy inventory management is determining the costs included in the total expected variable cost equation (K). In general, Navy inventory costs are difficult to determine. Recalling that A (ordering cost), C (item cost), π (time-weighted backorder cost) and I (inventory holding cost percentage) are included in the TVC equation, the only cost that can be accurately determined is item cost (C). Both inventory holding cost percentage and order cost are defined and used in the UICP model, but even these costs are merely estimates. For shipboard models

and the VOSL holding and ordering costs are nearly impossible to estimate and are not even considered. Time weighted backorder cost (π) is the most difficult to determine and is not estimated in any Navy models. In the UICP model π is included but the value of π is imputed from budget constraints and is not the actual shortage cost. Backorder costs are not explicitly considered in either the VOSL model or the shipboard models.

(2) Unknown Demand Distributions:

To determine optimum order quantities and reorder points using analytically determined equations in a stochastic inventory model, the usual procedure is to make certain assumptions about the distribution of lead time demand. Unfortunately, demand patterns for many Navy inventory items are extremely erratic and difficult to be described by a particular probability distribution. Additionally, computational constraints require that gross assumptions about distributions be made by item category.

One of the purposes of Ref. (5) was to examine the distributions used by the UICP model, namely the normal, negative binomial and Poisson distributions, along with several other distributions to determine their "goodness of fit" to Navy demand data. The study used the one sample Kolmogorov-Smirnov test, which provides a measure of goodness-of-fit of the

actual demand patterns to the candidate theoretical probability distributions. In addition to the three theoretical distributions already mentioned, the following were also tested: logistic, La Place, gamma, and uniform. The analysis of 610 items from the data base at Naval Air Station (NAS) at Brunswick produced the following results:

Items included	The percentage of line items for which the most accurate theoretical distribution provided an accurate fit to demand data at the 90% confidence level
All items (sample size = 610)	23%
Slow movers (sample size = 223) (Ave. quarterly demand $\leq$ 0.25)	28%
Medium movers (sample size = 355) (0.25 < ave. qtrly. demand $\leq$ 20.0)	28%
Fast movers (sample size = 32) (ave. qtrly. demand > 20.0)	78%

The study in ref. (5) concluded that the Navy demand patterns are very poorly described by the standard distributions. Those test results confirm what appears obvious from the erratic nature of Navy demand data.

(3) Unknown Demand Distribution Parameters:

In addition to the assumptions about demand distributions the parameters of the distribution must also be estimated. Usually, some method for forecasting the parameters for the upcoming period is used. The UICP model uses exponential smoothing to obtain current parameter estimates. The point is that the parameters are estimated separately from the assumptions about the distributions and the optimization equations, a questionable procedure.

(4) Computational Problems:

Time-weighted shortage-cost models, as opposed to the simpler cost per backorder type, are required by the Department of Defense for the Navy's UICP inventory. In fact, all top level, or wholesale, Department of Defense inventory systems are required by DOD Instruction 4140.39 to consider time-weighted, essentiality-weighted requisitions short as the measure of effectiveness. In Chapter II the equations to solve for Q* and r* were derived, and it was shown that Q* and r* each depended in a complex way on the other. For the Navy's inventory control points that manage up to approximately 500,000 items, it is not possibie to use the optimum equations for Q and r. The ICP's must use a separate equation to determine Q, and then use an approximation to the optimal risk equation to find r. Finally, a

computational problem of a different type has, until recently, existed on non-automated ships. Without computer assistance ashore these ships have simply been unable, with available personnel, to operate anything but a simple fixed-months-supply inventory system. This system does not use any quantitative analysis and probably produces less than optimal inventory policies.

(5) Constraints:

The analytical optimization technique described in the previous chapter assumes that the minimum cost inventory policy can be carried out. That is, it is assumed that the optimum policy is not affected by active constraints. In private business, a company can budget more money to the inventory as required. In the Navy, this is not usually the case and, in fact, budget and/or manpower constraints are always active. The UICP model imputes a sufficiently low backorder cost so that "affordable" reorder points result. The VOSL model is limited to 2.5 months average on-hand inventory. This 2.5 months inventory is divided between safety and operating stocks, thereby setting artificial financial constraints on both. Tight funding nearly always constrains shipboard inventory policies, particularly in the recent period of fiscal austerity.

This chapter has described some of the problems involved in using the standard inventory theory in Navy models. Since these problems prohibit using the exact theoretical models, the goal of the inventory manager should be to use the theory and information available to him to establish the best possible feasible policy. The next chapter describes the four basic Navy models: the UICP model, VOSL — the stock point model, and both the automated and non-automated AFLOAT models. In none of the models is traditional theory applied without modification although an attempt is made to merge traditional theory with feasible operating policies.

IV. CURRENT NAVY INVENTORY MODELS

This chapter describes, evaluates, and in some cases makes recommendations for improving the current Navy inventory models.

A. UNIFORM INVENTORY CONTROL PROGRAM

The UICP model is based on the stochastic demands, time-weighted backorder cost model. From Chapter II the total variable cost equation is

$$TVC = \frac{\lambda A}{Q} + IC[r + \frac{Q}{2} - \mu] + \frac{\pi}{S} B(Q,r)$$

where

$$B(Q,r) = \frac{1}{Q} \int_r^{\infty} (x - r)[F(x + Q;L) - F(x;L)]dx$$

and F(x;L) is the cumulative distribution of lead time demand. Now, to conform to the UICP model, B(Q,r) is included for exactness in the holding cost term and the backorder cost term is essentiality weighted, giving:

$$TVC = \frac{\lambda A}{Q} + IC[r + \frac{Q}{2} - \mu + B(Q,r)] + \frac{\pi E}{S} B(Q,r)$$

where

E = Item essentiality weight.

As before, taking the derivative with respect to r and setting it equal to 0 yields the following equation:

$$\text{(IV.1)} \qquad \int_r^{\infty} [F(x + Q;L) - F(x;L)dx = \frac{SQIC}{SIC + \pi E}$$

Using the theoretical model the proper procedure would be to solve for Q and r, using Equation (IV.1) above ($\frac{\partial TVC}{\partial r} = 0$) and ($\frac{\partial TVC}{\partial Q} = 0$). However, the following difficulties prevent using this model at the inventory control points:

(1) The entire procedure assumes that the time-weighted, backorder cost (π_i) for each item is known. In fact, π_i is unknown.

(2) In order to solve for reorder points from the optimum risk equation the distribution of lead time demand must be known. Chapter III described the analysis of Navy demand data in Ref. (5) which indicated that the data did not fit any of the standard distributions.

(3) The theoretical model assumes that there are no active constraints. In fact, either manpower or funding limitations are always constraints in actual practice.

(4) Even if the above three problems did not exist the iterative computations required to solve the two equations for Q* and r* repeatedly for each item in the inventory would be prohibitive.

Due to the above problems the UICP model is adapted to make it feasible within Navy limitations. The algorithm used at Navy Ship's Parts Control Center is as follows:

(1) Order quantities are computed separately from the TVC equation. No iterative computing is required, making the procedure computationally feasible. Specifically:

$$Q_i = \text{MIN}[5\lambda_i, \max(\sqrt{\frac{2\lambda A}{IC}}, 1, \frac{\lambda_i}{4})]$$

(basically, Q_i is the Wilson EOQ, constrained on the lower side by 1 and $\frac{\lambda_i}{4}$ [to satisfy manpower constraints], and on the upper side by $5\lambda_i$).

(2) Note that Equation (IV.1) above is very difficult to solve for r, since the optimum Q in that equation is also unknown. At this point an approximation technique (see Ref. (6)) is used to obtain the following risk equation currently used by SPCC:

$$\frac{\frac{\lambda_i}{4} IC_i}{\frac{\lambda_i}{4} I\, C_i + \pi\, W_i\, E}$$

(where W is the quarterly requisition frequency). This equation still contains π which is unknown. Therefore, a starting value for π is chosen, and the risk for each item is determined.

(3) Once the risk for each item is known, reorder points could be determined if the distribution of lead time demand were known. This problem is overcome by assuming that the

demand for all items is fit by the Poisson distribution (slow movers), the negative binomial distribution (medium movers), or the normal distribution (fast movers).

(4) If the funds available are sufficient, the Q's and r's are taken to be those determined as described above. If funds are insufficient, the model returns to step (2), this time with a lower value of π. Note that successive iterations reduce reorder points but never order quantities. The procedure continues to iterate, returning to step (2) until funding requirements are feasible. This iterative procedure "solves" the remaining two problems which made the theoretical model infeasible: the unknown π figure is "imputed" or forced by funding constraints and the final levels (Q_i, r_i) are automatically feasible with respect to the budget limitation.

Having seen how UICP procedures start with the basic theory and finally, after several changes, arrive at a completely different but feasible model, the next section examines the VOSL, or the stock point model. The VOSL model also uses various aspects of the inventory theory described in Chapter II in its derivation.

B. VARIABLE OPERATING AND SAFETY LEVEL (VOSL) – THE STOCK POINT MODEL

The VOSL model is used by the Navy stock points. VOSL was developed by Navy Fleet Material Support Office (FMSO) which provides the outside computer assistance required by the stock points to operate the VOSL. The model was developed in 1965 (see Ref. 11). The original VOSL model attempted to

determine safety levels with the goal of maximizing the dollar value of sales. Since 1965 several changes have been made to the model with the major change being a conversion to the "requisitions short" criteria to determine safety levels (see Ref. 4), and the current model purports to minimize requisitions short. The VOSL model is not based directly on the traditional inventory theory, although some aspects of the theory are used in the derivation. The traditional EOQ equation is used indirectly to figure order quantities, and a variation of the total variable cost equation together with the Lagrange technique and the concept of optimal risk form the basis of reorder point determinations.

The primary purpose of this section is to provide a clear, concise explanation of the VOSL model. For simplicity the model equations are not derived. The interested reader is referred to references 3, 4, and 11 for the derivations. It is hoped that this section will help the reader understand the basis of the VOSL model and perhaps spur future research into possible improvements.

1. VOSL Order Quantities

When VOSL was first implemented in 1965, computer and manpower constraints made the determination of individual order quantities for each item in the large, multi-item stock point inventories infeasible. Therefore, VOSL assigned each item to one of the following ten classes which in turn determined the order quantity in terms of months of demand. This practice continues today.

CLASS	Q(MD) = Q in units of months of demand
I	1.0 Month
II	1.5 Months
III	2.0 Months
IV	2.5 Months
V	3.0 Months
VI	4.0 Months
VII	5.0 Months
VIII	6.0 Months
IX	8.0 Months
X	12.0 Months

All items in the same class have identical order quantities in terms of months of demand. For example,

given $(\lambda_i) = 48$ and ITEM CLASS = V, $Q_i = \frac{48}{12} \times 3.0 = 12$ Units;

given $(\lambda_j) = 24$ and the same item class, $Q_j = \frac{24}{12} \times 3.0 = 6$ Units

Since the stated objective of VOSL is to minimize requisitions short subject to a budget constraint, it would seem logical to assign higher cost items smaller order quantities in terms of months of demand than lower cost items. In other words,

$$Q(MD)_i < Q(MD)_j, \quad \text{IF} \quad C_i > C_j$$

VOSL actually determines the order quantities based on the value of annual demand (VAD). Items with larger VADs are assigned smaller order quantities in terms of months of demand than items with smaller VADs. In other words,

$$Q(MD)_i < Q(MD)_j \,, \quad \text{IF} \quad VAD_i > VAD_j \,.$$

Specifically, since $\lambda_i C_i$ is the VAD for item i, the VADs for the boundary items between each of the ten classes of items are determined as follows:

$$\text{(IV.2)} \qquad \lambda_i C_i = [6P(\frac{Q_{i+1}(MD) + Q_i(MD)}{Q_{i+1}(MD) \cdot Q_i(MD)})]^2$$

(The inventory variable P will be explained below.) For example, if P = 1.5, the boundary in terms of VAD between class V and class VI would be

$$[6 \cdot 1.5(\frac{4.0 + 3.0}{4.0 \cdot 3.0})]^2 = \$27.56$$

Similarly, the boundary between class VI and class VII would be

$$[6 \cdot 1.5(\frac{5.0 + 4.0}{5.0 \cdot 4.0})]^2 = \$16.40$$

For this example all inventory items with a VAD between \$16.40 and \$27.56 would be assigned to class VI, and the

order quantities would be determined by

$$Q_i = \frac{\lambda_i}{12} \times 4.0$$

Equation (IV.2) above includes the inventory variable P, which in turn depends on α, the average operating level investment. Because of limitations imposed by higher authorities, VOSL is allowed an average total on-hand inventory investment of 2.5 months of demand. This inventory investment is divided between average operating level investment (α months of demand) and safety level stock ($2.5 - \alpha$ months of demand). Once management assigns a value to α, the inventory variable P is found as follows:

$$\text{(IV.3)} \qquad P = \frac{\alpha}{6} \frac{\sum_{i=1}^{n} C_i \lambda_i}{\sum_{i=1}^{n} \sqrt{C_i \lambda_i}}$$

Intuitively, the higher the value assigned to α the higher will be the resultant P from equation (IV.3). Consequently, the higher P value will result in larger VAD boundary points from equation (IV.2). This in turn will result in more items being assigned to higher VAD classifications and larger overall order quantities, or a larger average operating level investment.

2. <u>VOSL Reorder Points</u>

The current "requisitions short" VOSL model for setting reorder points was developed in 1968 (see Ref. 4).

The objective of the current VOSL in setting reorder points is to minimize requisitions short. VOSL is constrained to $(2.5 - \alpha)$ months of demand for safety level stock. The following two equations are used to determine reorder points:

$$\text{(IV.4)} \qquad RISK_i = \theta' \frac{(C_i)(Q_i)}{H_i}$$

where H_i is the number of customer requisitions for item i per year and θ' is the value of the Lagrange multiplier.

$$\text{(IV.5)} \qquad \theta' = \frac{1}{\left(\frac{2.5 - \alpha}{12}\right)} \sum_{i=1}^{n} \frac{(RISK_i)(H_i)(SL_i)}{(C_i)(\lambda_i)(Q_i)}$$

where SL_i is the safety level for item i. Since $RISK_i$ is a function of θ' and θ' is a function of $RISK_i$ (as well as SL_i which is also determined by RISK), an iterative procedure must be used. This procedure is described by the following algorithm:

(1) An initial θ' value is chosen. Then each $RISK_i$ is determined using equation (IV.4).

(2) Each SL_i is determined from the corresponding $RISK_i$ (this procedure is explained below).

(3) The sum of safety level investment is computed, i.e., $\sum_{i=1}^{n} C_i SL_i$. The algorithm requires that the funds spent on safety level stock be within a specified small value of the safety level budget constraint.

Defining the specified small value as epsilon the following question is considered:

$$\text{Is} \quad \left| \sum_{i=1}^{n} C_i SL_i - \left(\frac{2.5 - \alpha}{12}\right) \sum_{i=1}^{n} C_i \lambda_i \right| \leq \text{Epsilon ?}$$

If yes, then stop.

(4) If not, compute a new value of θ' using equation (IV.5) and the current values for $RISK_i$ and SL_i.

(5) Go to (1) with the new value of θ'. Continue until the safety level investment is within the epsilon value of funds available in step (3).

To provide an intuitive explanation of how the algorithm works, it is worthwhile to consider what happens when the initial chosen value of θ' is too large. In steps (1) and (2) the RISK and safety level for each item are determined. Since θ' is larger than the feasible θ' value, the initial RISKs are "too large" and the corresponding safety levels "too small." Since the safety levels are below feasible values, the funds spent on safety stocks will not be within epsilon of the funding limitation. Therefore, the algorithm will proceed to step (4) where the new θ' value is computed. The new θ' value is computed from the current values of RISKs and safety levels, and will be smaller than the initial θ' value. The algorithm will continue in this manner until the termination condition in step (3) is met. At each

successive iteration, the θ' value becomes smaller while the safety levels become larger. (If the initial θ' value is too small rather than too large, θ' will increase at each iteration, while the safety levels decrease.) In practice, the convergence is rapid and the computer time required is not excessive.

To compute the safety level for item i from $RISK_i$, the normal distribution is assumed for lead time demands. The standard deviation of lead time demand (σ) is obtained from the following equation:

$$\sigma_i = 1.25(MAD_i) \cdot \sqrt{\frac{\text{LEAD TIME}_i}{3}} \tag{IV.6}$$

where MAD is the average quarterly mean absolute deviation and the lead time is in units of months. Safety stock is then determined by

$$\text{SAFETY LEVEL}_i \; (SL_i) = \sigma_i \cdot z$$

where z is the appropriate standard normal deviate corresponding to $RISK_i$.

Once SL_i is known, the actual reorder point is found by

$$r_i = SL_i + \text{LEAD TIME STOCK}_i$$

where

$$\text{LEAD TIME STOCK}_i = \text{AVG LEAD TIME IN MONTHS X AVG MONTHLY DEMAND}$$

To illustrate how reorder points are determined, consider an item with the following characteristics:

AVERAGE LEAD TIME = 1 MONTH

MAD = 12.5 UNITS

AVERAGE MONTHLY DEMAND = 20 UNITS PER MONTH

ITEM CLASS III, Q = 20 X 2 = 40 UNITS

UNIT PRICE (C) = $.50

ANNUAL CUSTOMER REQUISITION FREQUENCY (H)

= 100 REQUISITIONS

θ' (FROM ALGORITHM) = .90

LEAD TIME STOCK is:

1 MONTH X 20 UNITS PER MONTH = 20 UNITS

SAFETY STOCK COMPUTATION:

$$\text{RISK} = \theta' \frac{(C_i)(Q_i)}{H_i} = .90 \frac{(.50)(40)}{100} = .18$$

$$\sigma = 1.25\ (12.5)\ \sqrt{1/3} = 9.02$$

z = 0.915 (THE STANDARD NORMAL DEVIATE CORRESPONDING TO A RISK OF .18)

SAFETY LEVEL = 0.915(9.02) = 8.25 $\simeq$ 9 (ROUNDED UP)

REORDER POINT = 20 UNITS + 9 UNITS = 29 UNITS

3. Evaluation of VOSL

Like any other model, VOSL is best evaluated by the inventory managers who use it, i.e., how well does it actually work for the stock points? Before addressing this question, it appears that the method of determining order quantities could be improved. The order quantity method, whereby all items are assigned to one of ten classes with all items in the same class having the same order quantity in terms of months of demand, was designed in 1965 and has not changed since then. With today's improved computer capability the possibility of determining order quantities directly by a variant of the EOQ equation should be investigated (i.e., $Q_i = K\sqrt{\frac{\lambda}{C}}$). The inventory variable K could then be used as a management tool to be varied according to the availability of procurement funds. This would give management a direct connection between fund availability and inventory levels. Additionally, the assumption that demand data is fit by the normal distribution is questionable, particularly since Navy demand data in general is very erratic and does not conform to theoretical distributions. An investigation of the demands made on stock points should be done to clarify this issue.

The major shortcoming of VOSL is that there is no direct coordination between the financial and inventory managers built into the model. Ref. (8) points out that while the VOSL stocking policy is based on a 2.5 month average inventory investment, funding grants depend on the sales

(total satisfied demands) of prior years. There is no direct connection between these two factors. When a stock point is short of funds demands cannot be filled, thereby reducing sales and the funds for the next period. A "vicious circle" can develop where the stock points with the greatest need for funds to make up for past deficiencies are continually reduced in funding. Ref. (8) stresses that VOSL does not work in actual practice as it was designed. In fact, stock point inventory managers devise local "ad hoc" decision rules to survive from one period to the next. Some managers may reduce all VOSL order quantities by a certain percentage, others may place a monetary limit on resupply requisitions, etc.

The solution to this problem is very difficult. Certainly, higher funding levels for the stock points would help. However, today's fiscal austerity makes this solution extremely unlikely. Ref. (1) appears to offer the most promising alternative to VOSL for stock point inventory management and should be investigated for possible implementation. Unlike VOSL, the model prescribed in Ref. (1) allows the inventory manager to exercise dynamic control over the inventory, and inter-item competition for limited funding is considered as an input to stocking decisions.

The remaining models to be discussed are the AFLOAT models which, due to the inherent limitations operative in afloat inventory systems, are considerably simpler than the

models already discussed. The next model discussed is the Shipboard Uniform Automated Data Processing System (SUADPS).

C. AUTOMATED AFLOAT

This section describes the current inventory procedures utilized aboard automated ships, or SUADPS. This section does not distinguish between the SUADPS-207 model used by supply ships and the SUADPS-END USE (SUADPS-EU) model used by aircraft carriers and other large ships. Although there are some minor differences, the points of discussion are applicable to both and the analysis can be greatly simplified by addressing the automated afloat model in general.

Chapter II described the basic Wilson, EOQ equation,

$$Q = \sqrt{\frac{2\lambda A}{IC}}$$

Despite its simplicity, a variation of this model is used on automated ships. Specifically, the EOQ is computed for each individual item using the following:

$$EOQ = KEOQ\sqrt{\frac{\lambda}{C}}$$

where:

EOQ = The economic order quantity

KEOQ = A management control which considers the availability of funding and the capability of shipboard personnel to handle the workload associated with the resupply orders.

It can be seen that

$$\sqrt{\frac{2\lambda A}{IC}} = \sqrt{\frac{\lambda}{C}} \cdot \sqrt{\frac{2A}{I}}$$

or

$$KEOQ = \sqrt{\frac{2A}{I}}$$

So in effect the model combines the unknown costs, A and I, into a constant which is used as a management tool.

There are two other management controls used in the model: MINEOQ and MAXEOQ. If the computed EOQ is less than the MINEOQ or greater than MAXEOQ, the order quantity is revised upward or downward to the MIN or MAX. In other words, order quantity equals EOQ if MINEOQ $\leq$ EOQ $\leq$ MAXEOQ, but can never be outside this range. These additional controls prevent extremely high or low order quantities, particularly for very low cost items for which the factor $\sqrt{\frac{\lambda}{C}}$ and consequently Q can be very large.

The reorder points are determined by a fixed months supply procedure. First an order and shipping time (OST) in months is obtained, based mainly on past lead times. Then order and shipping time quantity (OSTQ) is determined as follows:

$$OSTQ = OST \cdot \frac{\lambda}{12}$$

Then the safety stock quantity (SSQ) is determined:

$$SSQ = K_{ss} \cdot \frac{\lambda}{12}$$

where

K_{ss} = A management control which expresses in months the desired safety stock policy.

Finally, the reorder point is:

$$r = OSTQ + SSQ$$

The overall evaluation of this model is favorable. No attempt is made to define and use unknown or hard to predict costs. By eliminating the unknown costs, A and I, from the Wilson equation and using instead the constant factor, KEOQ, $\times \sqrt{\frac{\lambda}{C}}$, only known factors are used. To set order quantities the $\sqrt{\lambda/C}$ factor alone provides larger order quantities for high demand, low cost items, which is intuitively appealing. Also, the research based on simulation work conducted by the author on non-automated ships indicates that the same model, i.e., $Q = \text{constant} \cdot \sqrt{\lambda/C}$, provides excellent results. Additionally, this technique enables management to use the control KNOB, KEOQ, to vary order quantities depending on the resources available so that the selected policy is feasible within budget and manpower constraints. Finally, this model is simple enough to be easily implemented within the computer capabilities of these ships.

The major shortcoming of this model appears to be in the fixed-months-supply reorder point determination. Different methods for setting reorder points should be explored. One possibility is a periodic review model based on the mixed Bernoulli/exponential distribution and Lagrange theory as recommended in Ref. (12). Another possibility is a model similar to that proposed for non-automated ships in Chapter VI. In any case it would appear that continued use of the modified Wilson EOQ to determine order quantities along with an improved method to set reorder points should be explored.

So far this chapter has discussed the UICP, VOSL, and automated afloat models. The final model to be discussed is the simplest Navy Inventory model — the non-automated afloat model.

D. NON-AUTOMATED AFLOAT MODEL

Currently non-automated ships use a fixed-months-supply inventory system for items stocked because of usage. The specific inventory levels depend on ship type, location, etc., but in most cases levels are set as follows:

$$\text{HIGH LEVEL (H/L)} = 4.5 \cdot \frac{\lambda}{12}$$

$$\text{REORDER POINT} = r = 3.5 \cdot \frac{\lambda}{12}$$

$$Q = H/L - r$$

This simplistic model does not attempt to apply any quantitative technique to inventory management. The author's detailed recommendation for non-automated ships along with the results of the supporting research is the subject of Chapter VI.

This chapter has described the four basic Navy inventory models currently being used. The next chapter proposes a new "approach" to Navy inventory management which could eliminate some of the shortcomings of the present models.

V. A NEW APPROACH TO NAVY INVENTORY MANAGEMENT

Most past attempts to improve upon Navy inventory models have concentrated on changes within the current framework. It may be more productive to examine completely different models that would do the following:

(1) The models would be based only on information that can be obtained with a certain degree of accuracy. Unknown costs would not be used in the derivation of the model. Demand distributions and distribution parameters would also not be used unless they accurately describe the real world situation.

(2) All information that could be predicted with reasonable accuracy would be used in the model if it results in improved inventory management.

(3) The objective in determining reorder points and order quantities would be clearly defined and the model would be derived so that the decision variables are optimal with respect to the stated objective.

The UICP model is derived from the total variable cost equation:

$$TVC = \frac{\lambda A}{Q} + IC[r + \frac{Q}{2} - \mu] + \frac{\pi}{S}[\frac{1}{Q} \int_{r}^{\infty} (x-r)(F(x+Q;L) - F(x;L)]dx$$

For this equation to be useful in providing decision variables that minimize costs, all the factors must be known. If just one factor is unknown, the results of using the model may not be optimal. It has been previously pointed out that numerous factors in the TVC equation are unknown, and Chapter IV describes how the current UICP model "adapts" to the unknown factors. Unfortunately, the result is an "ad hoc" inventory model that may not be optimal.

On the other hand, the current non-automated afloat model fails to use information that can significantly improve inventory management. This is verified by the results of the computer simulation work done by the author and reported in the next chapter. The model presented in the next chapter is also based on a clearly defined objective, i.e., to minimize unsatisfied customer requisitions subject to a constraint on average inventory investment. Many models attempt to equalize risk (the probability of a stockout during a reorder period) for all inventory items. This does not appear to be a realistic goal of an inventory system.

Applying the new "approach" to the UICP model results in much simpler inventory methods than presently exist. Since the only parameters that can be accurately predicted are λ and C, the economic order quantity equation would determine order quantities proportional to the square root of the ratio of the rate of demand to the unit cost (i.e.,

$$Q_i = \text{Constant} \cdot \sqrt{\frac{\lambda_i}{C_i}} \;).$$

This allows management to vary the constant depending on the availability of funds. Thus financial constraints would reduce order quantities and not reorder points, as is the case in the present UICP model. To determine reorder points the objective of management must be clearly defined. Further study would have to be made into the distribution of lead time demand. This study should consider compound probability distributions as possibilities to model ICP demand data. If an accurate distribution were not found, actual reorder point calculations could be done using a non-parametric approach. The reader is referred to Ref. (9) for an inventory model that follows this approach.

The final chapter describes the application of the new approach to inventory management on non-automated ships. It includes the results of simulation work done at Navy Fleet Material Support Office and is the principal research effort of this thesis.

VI. AN IMPROVED INVENTORY MODEL FOR NON-AUTOMATED SHIPS

This final chapter presents the author's recommendation to improve non-automated shipboard inventory management. The current model is presented first, followed by the model proposed by the author. A short explanation of the computer program used to evaluate the models via computer simulation is then presented. The measures of effectiveness (MOEs), and the comparative statistics of the present and proposed models are then addressed. This is followed by the author's specific recommendation.

A. PRESENT MODEL

At the present time non-automated ships use a fixed-months-supply method to determine both order quantities and reorder points for items stocked due to usage (usually called SIM* items). The order quantities are taken to be one month of stock, and the reorder points are taken to be three and one-half months of stock.

$$r = 3.5 \times \frac{\lambda}{12}$$

$$Q = \frac{\lambda}{12} \quad \text{(one month of demands)}$$

---

*Selected Item Management

Originally, this simplistic method of setting levels was adopted for non-automated ships because of computational difficulties in using more complicated, and perhaps more effective, inventory procedures. Now, however, this is no longer a valid reason with the availability of inexpensive and portable computers.

B. PROPOSED MODEL

The possibility of using computers to support the ships allows more sophisticated and effective techniques to be used. The proposed model determines reorder points that minimize unsatisfied customer demands within a budget constraint on average inventory. Order quantities are determined using an economic-order-quantity equation.

1. Reorder Points

The determination of reorder points requires knowledge of the probability distribution for lead time demand. Therefore, lead time demand was analyzed for an accurate fit. A technique to minimize customer requisitions short subject to a constraint on average inventory investment was used to solve for reorder levels.

The data available for this part of the study was two years of actual demand history for five ships. Since items stocked due to usage (SIM items) generally require four demands per year to qualify for stocking and two demands per year to retain qualification, only those items that experienced at least six demands in the observed two year period were

selected. Two hundred such items were picked at random for the analysis. Since the analysis requires a distribution for lead time demands, the demand quantities were segmented into thirty day periods. The lead time was taken to be thirty days since stateside lead times for orders placed to supply centers are normally considered to be one month. Furthermore, deployed ships are usually replenished once a month with reorders for each replenishment delivered during the prior replenishment. Thus for items carried by the supply centers and the replenishment ships, the thirty days lead time assumption should be adequate. Since nearly all items stocked due to usage on any ship are common usage items throughout the Navy and are therefore almost always stocked by the supply centers and supply ships, the lead time assumption should apply to all items of interest in this thesis.

One characteristic of the data that was immediately apparent once the demand quantities were segmented into 30 day periods was the large number of months (out of the 24 months) during which no demand occurred. This high incidence of zero observations made it apparent the demand distribution would have to reflect this high probability mass at zero. In fact, further analysis showed that the average number of months with non-zero demand quantity for the 200 line items was 5.405 months out of 24. The distribution of the number of non-zero demand observations for the 200 items is shown on the following page. This initial data analysis suggested if a distribution fit were to be found it would be a compound

```
                FREQUENCY  (number of line items
                            experiencing N non-zero
                            demand months out of 24)

N
*
4   xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx                  49

5   xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx   64

6   xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx       60

7   xxxxxxxxxxxxxxxxxxx                                                19

8   xxxx                                                                4

9   xxx                                                                 3

10

11

12

13   x                                                                  1

14

15

16

17

18

19

20

21

22

23

24

*  Only line items for which N ≥ 4 were considered
```

distribution, with one process determining whether a demand would occur and the other determining the quantity of demand given that it occurs. (See ref. (12) for a similar distribution applicable to supply ships.) The former process can be modelled as a Bernoulli process with parameter ρ, where ρ is the probability that a positive demand occurs. The one sample Kolmogorov-Smirnov goodness-of-fit test was then used to evaluate various standard probability distributions against the non-zero demand quantities. The following distributions were evaluated: geometric, negative binomial, Poisson, exponential, normal, and log-normal. Of these, the geometric distribution provided the most accurate fit to the data. Therefore, the compound Bernoulli-geometric distribution was used to model lead time demand. The cumulative distribution function, where ρ is the probability of a positive demand and p is the geometric distribution parameter, is given by the following:

$$
\begin{aligned}
F(x) &= (1-\rho) + \rho \sum_{k=1}^{x} pq^{k-1} \\
&= (1-\rho) + \rho p(1 + q + \dots + q^{x-1}) \\
&= (1-\rho) + \rho p\left(\frac{1 - q^{x}}{1 - q}\right) = 1 - \rho + \rho(1 - q^{x}) \\
&= 1 - \rho q^{x}
\end{aligned}
$$

Therefore,

$$p[X > r] = 1 - F(r) = \rho q^r$$

The objective of the model is to minimize customer requisitions short per year subject to a budget constraint on average inventory investment. An expression for customer requisitions short for the Bernoulli-geometric distribution must be derived. The expected number of units short per cycle is

$$\sum_{x=r+1}^{\infty} \rho p q^{x-1}(x-r) = \rho p(q^r + 2q^{r+1} + 3q^{r+2} + \ldots)$$

$$= \rho p q^r(1 + 2q + 3q^2 + \ldots)$$

$$= \rho p q^r \cdot \frac{d}{dq}\left(\frac{q}{1-q}\right) = \frac{\rho p q^r}{(1-q)^2}$$

$$= \frac{\rho q^r}{p}$$

The expected number of cycles per year equals

$$\frac{\lambda}{Q} = \frac{12 \cdot \rho}{Q \cdot p}$$

To determine the expected number of units short per year, define Z_i as the number of units short in cycle i. Then the number of units short per year is

$$Z_1 + Z_2 + Z_3 + \ldots + Z_N$$

where N is the number of cycles per year. Appealing to renewal theory, the expected number of units short per year is

$$E[\text{UNITS SHORT/YR}] = E_N[E(\text{UNITS SHORT/YR}|N)]$$

$$= E_N[nE(Z_i)] = E[N]E[Z]$$

$$= \frac{12 \cdot \rho^2 \cdot q^r}{p^2 \cdot Q}$$

The expected number of customer requisitions short per year is the above expression divided by the average requisition size, S, or

$$E[\text{CUSTOMER REQUISITIONS SHORT/YR}] = \frac{12 \cdot \rho^2 \cdot q^r}{p^2 \cdot Q \cdot S}$$

An expression for average inventory investment must now be derived. Since the inventory position (IP) is defined to be the quantity on hand (OH) plus the quantity on order (OO) minus the quantity backordered (BO), then the expected quantity on hand equals E(IP) - E(OO) + E(BO). The expected inventory position is Q/2 + r. The expected quantity on order equals μ, the expected lead time demand, or E(OO) equals $\frac{\rho}{p}$. For the expected backorders term let y be the number of backorders. The state probability $\psi(y)$ that there are

y backorders at an arbitrary time t (see ref. 7) is

$$\psi(y) = \frac{1}{Q}\sum_{j=1}^{Q} f(y + r + j) = \frac{1}{Q}\sum_{u=y+r+1}^{y+r+Q} f(u)$$

$$= \frac{1}{Q}(\rho q^{y+r} - \rho q^{y+r+Q}) = \frac{\rho}{Q}(q^{y+r} - q^{y+r+Q})$$

The expected quantity backordered is given by $\sum_{y=0}^{\infty} y \cdot \psi(y)$, so that

$$E(BO) = \frac{\rho}{Q}\sum_{y=0}^{\infty}(q^{y+r} - q^{y+r+Q})y$$

$$= \frac{\rho}{Q} \cdot q^{r+1} \cdot (1-q^{Q}) \cdot \sum_{y=0}^{\infty} y \cdot q^{y-1}$$

$$= \frac{\rho}{Q} \cdot q^{r+1} \cdot (1-q^{Q}) \cdot \sum_{y=0}^{\infty} \frac{d}{dq} q^{y}$$

$$= \frac{\rho}{Q} \cdot q^{r+1} \cdot (1-q^{Q}) \cdot \frac{d}{dq}\left(\frac{q}{1-q}\right)$$

$$= \frac{\rho \cdot q^{r+1} \cdot (1-q^{Q})}{Q \cdot (1-q)^{2}} = \frac{\rho \cdot q^{r+1} \cdot (1-q^{Q})}{Q \cdot p^{2}}$$

Combining terms, we have

$$E(OH) = E(IP) - E(OO) + E(BO) ,$$

or

$$E(OH) = r + \frac{Q}{2} - \frac{\rho}{p} + \frac{\rho \cdot q^{r+1} \cdot (1-q^{Q})}{Q \cdot p^2}$$

The problem can now be stated as follows:

$$\text{MINIMIZE} \quad \sum_{i=1}^{n} \frac{12 \cdot \rho_i^{\,2} \cdot q_i^{\,r_i}}{p_i^{\,2} \cdot Q_i \cdot S_i}$$

$$\text{SUBJECT TO} \quad \sum_{i=1}^{n} C_i \left(r_i + \frac{Q_i}{2} - \frac{\rho_i}{p_i} + \frac{\rho_i \cdot q_i^{\,r_i+1} \cdot (1-q_i^{\,Q_i})}{Q_i \cdot p_i^{\,2}}\right) \leq B$$

where B is the constraint for inventory investment. Forming the Lagrange equation gives

$$L(r_1, r_2, \ldots, r_n; Q_1, Q_2, \ldots, Q_n; \theta) = \sum_{i=1}^{n} \frac{12 \cdot \rho_i^{\,2} \cdot q_i^{\,r_i}}{p_i^{\,2} \cdot Q_i \cdot S_i}$$

$$+ \theta \left[\sum_{i=1}^{n} C_i \left(r_i + \frac{Q_i}{2} - \frac{\rho_i}{p_i} + \frac{\rho_i . q_i^{\,r_i+1} \cdot (1-q_i^{\,Q_i})}{Q_i \cdot p_i^{\,2}}\right) - B\right]$$

Taking the partial derivative with respect to Q_i and setting it to zero gives

$$\frac{\partial L}{\partial Q_i} = \left[-\left(\frac{12 \cdot \rho_i^{\,2} \cdot q_i^{\,r_i}}{p_i^{\,2} \cdot Q_i^{\,2} \cdot S_i}\right) + \frac{\theta C_i}{2} - \frac{\theta \cdot C_i \cdot q_i^{\,r_i+1} \cdot \rho_i}{Q_i^{\,2} \cdot p_i^{\,2}} \right.$$

$$\left. - \frac{\theta \cdot C_i \cdot q_i^{\,Q_i + r_i+1} \cdot \rho_i \cdot \ln q_i}{Q_i \cdot p_i^{\,2}} + \frac{\theta \cdot C_i \cdot q_i^{\,Q_i + r_i+1} \cdot \rho_i}{Q_i^{\,2} \cdot p_i^{\,2}} \right] = 0$$

The above equation does not yield a simple expression for Q_i. An iterative procedure would be required to solve for Q_i and this would be beyond the computer capabilities of non-automated ships for multi-item SIM inventories. Therefore, as with the UICP model order quantities were determined by a separate procedure described in the next section and the formulation at hand was used to determine reorder points only. Taking the partial derivative with respect to r_i, setting it equal to zero, and solving for r_i results in the following:

$$r_i = \frac{\ln \left[\dfrac{-\theta \cdot C_i \cdot Q_i \cdot p_i^2}{\rho_i^2 \cdot \ln q_i \left(\frac{12}{S_i} + \theta \cdot C_i (1 - q_i^{Q_i}) \cdot q_i \right)} \right]}{\ln q_i}$$

Unfortunately, this expression for r_i is also complex. Some inventory models eliminate the E(BO) term from the equation for E(OH) for ease of computations. DOD INST 4140.39 requires this to be done for wholesale military inventory models, with the justification that the E(BO) term has little effect on the resultant decision rules. For the model under consideration this will be true only if the funds available for inventory investment (B) are sufficient to result in reasonable reorder points and order quantities. Then the E(BO) term, $\frac{\rho \cdot q^{r+1} \cdot (1-q^Q)}{Q \cdot p^2}$, will be small and will not significantly affect the results of the model. The decision as to whether to eliminate the E(BO) term must be made by management, considering the

trade-off between the computational savings and the reduced accuracy of the model at the given budget figure. If the E(BO) term is dropped, the expression for r_i becomes

$$(1) \qquad r_i = \frac{\ln\left[-\left(\dfrac{\theta \cdot C_i \cdot p_i^2 \cdot Q_i \cdot S_i}{12 \cdot \rho_i^2 \cdot \ln q_i}\right)\right]}{\ln q_i}$$

Equation (1) was used in the computer simulation model to determine reorder points.

2. Order Quantities

The proposed order quantity equation is based on the "Wilson" EOQ equation.

$$Q = \sqrt{\frac{2\lambda A}{IC}}$$

Since only λ and C can be estimated, the actual equation used is the following:

$$(2) \qquad Q = M_1 \times \sqrt{\frac{\lambda}{C}}$$

To prevent extremely large order quantities, Q can not exceed $M_2 \times \lambda$. M_1 and M_2 are management controls that can be varied depending on the availability of funds and personnel. This method of determining order quantities is nearly identical to that used on automated ships for several years under the Shipboard Uniform Automated Data Processing (SUADPS) model.

C. THE COMPUTER PROGRAM

The computer program used to compare the present and proposed models was the Alternative Cosal and SIM Simulator, written in SIMSCRIPT I.5. The simulator is a single item, single echelon model, meaning that it simulates one item at a time through the entire simulation period, during which statistics are generated and accumulated by year. Three years of actual demand data from a large, non-automated ship were used in the analysis. Since the simulator starts all items as non-SIM and it was estimated that it takes from one to one and a half years for the system to reach steady state, only the third year statistics are considered. This limitation was not considered serious, since the analysis still considered approximately 500 SIM items for a year of steady-state operation.

The program simulates all aspects of a shipboard inventory system. Each inventory item is considered individually for the entire three year period. The actual demand data are used, and once an item qualifies as SIM (four demands within a year) the rules that have been installed in the program for SIM management are used to determine order quantities and reorder points. Statistics are generated to be used in evaluating and comparing different SIM rules. Additional information on the use of the program can be obtained from Navy Fleet Material Support Office (CODE 9233).

D. MEASURES OF EFFECTIVENESS

Three measures of effectiveness were considered: SIM REQUISITION EFFECTIVENESS, NUMBER OF SIM RESUPPLY REQUISITIONS REQUIRED, and AVERAGE SIM INVENTORY REQUIRED.

1. SIM Requisition Effectiveness

This is the total percentage of SIM customer requisitions for which stock is available to satisfy, or

$$\text{Effectiveness} = \frac{\text{NR. OF CUSTOMER REQUISITIONS SATISFIED}}{\text{TOTAL NR. OF CUSTOMER REQUISITIONS PLACED}}$$

2. Number of SIM Resupply Requisitions

This is the total number of resupply requisitions submitted for all SIM items. This statistic is the most meaningful measure of the workload required to operate the inventory system, not only on-board the ships but also throughout the Navy supply system.

3. Ave SIM Inventory Required

This reflects the funding that would be required to be devoted to inventory of SIM items on-board the ships.

E. SIMULATION RESULTS

The initial simulation runs attempted to determine appropriate values for M_1 in equation (2) and M_2. The values $M_1 = 18$ and $M_2 = 1.5$ appeared to result in the best trade-off between better effectiveness/fewer reorders and inventory investment. After M_1 and M_2 were determined, the objective was to obtain a simulation run with the proposed model that

would result in an average inventory investment nearly identical to that obtained using the present model. An accurate comparison of the present and proposed models could then be made by considering the other measures of effectiveness. This would impute an appropriate value of θ in equation (1), the reorder point equation. In the actual simulation run, the imputed value of θ was .00125, with the interpretation being that at the applicable budget figure for average inventory investment an additional dollar invested in inventory would reduce customer requisitions short by .00125. A lower budget figure would increase θ, and vice versa. Also, as the budget is reduced and θ increases, the expected backorder term takes on added significance in the determination of reorder points. The following simulation runs comparing the present and proposed models with nearly identical average inventory figures were obtained:

	PRESENT MODEL	PROPOSED MODEL
SIM EFFECTIVENESS	79.50	85.62
# OF RESUPPLY ORDERS	1005	581
AVE SIM INVENTORY	42,087	41,528

The above statistics clearly show the superiority of the proposed model. For approximately the same inventory investment the new system eliminates about 30 percent $\left(\frac{20.5 - 14.38}{20.5}\right)$ of the unfilled customer requisitions and about 42 percent $\left(\frac{1005 - 581}{1005}\right)$ of the resupply orders.

F. SPECIFIC RECOMMENDATION/POTENTIAL BENEFITS

At the present time only some of the non-automated ships are supported by shore-based computers. Additional computer support could be provided to the ships by the use of small, dedicated mini computers on board the ships. These small computers would provide faster results to the inventory managers. The recommended system could be implemented on a representative number of the supported ships initially. It is anticipated that the proposal would not require a significant increase in computer time since it only replaces the fixed-months-supply level settings with the new procedures. After several months of operation on the test ships the system could be evaluated for possible implementation Navy-wide. The evaluation could be accomplished using the SIM effectiveness/ inventory reports that are already provided to the ships.

The parameters in the economic order quantity equation (equation (2)) could be set initially at the values used in the simulation run, i.e., $M_1 = 18$, $M_2 = 1.5$. These parameters could be varied based on the inventory and effectiveness statistics achieved. The value of the Lagrange multiplier, θ, in the reorder point equation (equation (1)) would be imputed from the budget constraint. It is anticipated that the budget constraint would be fairly constant for all ships of the same type. An appropriate budget constraint on inventory investment depending on the type ship could be used unless the ship's Supply Officer prefers to submit another

budget figure based on his ship's financial status. This would provide a direct interaction between fund availability and reorder levels, an additional benefit of the proposed model.

One other potential benefit of the proposed model should be noted. The significant reduction in resupply requisitions (42 percent based on the simulations) would have a positive effect on the Navy supply system. SIM resupply requisitions from non-automated ships account for a significant number of requisitions submitted to Navy supply activities. Reducing their number by 42 percent would save the Navy the processing and administrative costs associated with these requisitions.

LIST OF REFERENCES

1. Daeschner, W.E. Jr., "Models for Multi-Item Inventory Systems with Constraints," Ph.D. Thesis, Naval Postgraduate School, 1975.

2. Department of Defense Instruction 4140.39, July 17, 1970.

3. Fleet Material Support Office ALRAND Report 47A, "The Stock Point Inventory Levels Model (VOSL)," by Lt. A.P. Tully, SC, USN, 29 April 1969.

4. Fleet Material Support Office ALRAND Working Memorandum 144, "Variable Operating and Safety Level (VOSL) Requisitions Short Model," by P.C. Kohlhaas, 28 May 1968.

5. Fleet Material Support Office Report 120, "Probability Distributions for Leadtime Demand," by T. Tupper, 26 November 1975.

6. Fleet Material Support Office Report 82, "Procurement Cycles and Safety Levels of Supply for Secondary Items," by R.L. McNertney, et al, 24 August 1972.

7. Hadley, G. and Whitin, T.M., Analysis of Inventory Systems, Englewood Cliffs, N.J.: Prentice-Hall, 1963.

8. Lewis, R.E. and Perkins, R.D., "An Investigation of Variable Operating and Safety Level Stocking and Funding Policy," M.S. Thesis, Naval Postgraduate School, 1974.

9. Lippert, K.W., "An Analysis of the Ship's Parts Control Center Inventory Model and a Possible Alternative Model," M.S. Thesis, Naval Postgraduate School, 1977.

10. Richards, F.R., "Comments on the Distribution of Inventory Position in a Continuous-Review(s,S) Inventory System," Operations Research, Vol. 23, No. 2, March-April 1975.

11. Ruete, E.S., "The Variable Stock Level System of Inventory Management for Navy Shore Activities," Unnumbered Report, Navy Fleet Material Support Office, 11 August 1965.

12. Taylor, C.F. Jr., "A Multi-Item Inventory Model for Combat Stores Ships," M.S. Thesis, Naval Postgraduate School, 1975.

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